

Transport properties of the XX chain in a staggered magnetic field

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Abstract

The thermal and spin Drude weights of the XX spin chain in staggered and uniform magnetic fields are exactly calculated and the low temperature behaviours of these quantities are derived. The temperature dependence of the thermal Drude weight shows double-peak behaviour as a result of the energy gap caused by the staggered magnetic field. The magnetic Seebeck coefficient is also analyzed in detail to investigate the magnetothermal effect.

Key words: transport property, XX spin chain, Drude weight, magnetothermal effect, Seebeck coefficient

1. Introduction

Transport phenomena in one-dimensional quantum systems have attracted much interest since the ballistic (non-dissipative) transport behaviours of integrable quantum many-body systems were indicated [1,2]. In general three-dimensional systems, ballistic conduction appears only when the current is a constant of motion. On the other hand, in one-dimensional integrable systems, a non-conserved current is also considered to be ballistic as a consequence of a finite overlap with the conserved quantities underlying integrability [2].

Quantum spin chains are typical examples of one-dimensional integrable systems and they have been analytically and numerically investigated by many authors [1–14]. Moreover, experimental researches on magnetic materials described by one-dimensional or quasi one-dimensional spin-1/2 magnets have been reported. For instance, in experiments on the Heisenberg chain compounds SrCuO_2 and Sr_2CuO_3 , anomalously large thermal conductivities were reported [15,16]. This indicates a quasi-ballistic heat transport carried by spinons or magnons with a mean-free path that is much larger than the correlation length.

In the present study, we consider the XX spin chain in a staggered magnetic field. This quantum spin chain is an easily solvable model, therefore, it offers a good example of the system with ballistic conduction. The dispersion relation of the spinless fermion describing this system has an energy gap caused by the staggered magnetic field. To investigate the effect of this energy gap on the transport properties, we exactly calculate the thermal and spin Drude weights and analyze the magnetic Seebeck coefficient which is a criterion for clarifying the spin directions of carriers.

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2. Exact solution of the model

The XX spin chain in a staggered magnetic field is described by the following Hamiltonian:

$$\mathcal{H}_0 + \mathcal{H}_{\text{st}} = \frac{J}{2} \sum_{n=1}^{2N} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) - h_{\text{st}} \sum_{n=1}^{2N} (-1)^n \sigma_n^z, \quad (2.1)$$

where J is the coupling constant, h_{st} is the magnitude of the staggered field and $\sigma_n^{x,y,z}$ are the Pauli matrices acting on the n -th site. The total number of spins is assumed to be an even number $2N$, because the periodic boundary condition $\sigma_{n+2N}^{x,y,z} = \sigma_n^{x,y,z}$ is applied. In a uniform magnetic field $2h$, the Zeeman energy

$$\mathcal{H}_Z = -h \sum_{n=1}^{2N} \sigma_n^z \quad (2.2)$$

is added to eq. (2.1), where the total magnetization is $M = \sum \frac{1}{2} \sigma_n^z$.

It is easy to diagonalize the total Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{st}} + \mathcal{H}_Z \quad (2.3)$$

using the Jordan-Wigner transformation and Fourier transformation [17]. As a result, the above Hamiltonian (2.3) can be expressed in terms of the spinless fermion annihilation and creation operators c_q and $c_q^\dagger$, respectively, with the wave number q as follows:

$$\mathcal{H} = \sum_q \omega_q \left(c_q^\dagger c_q - \frac{1}{2} \right). \quad (2.4)$$

Here the dispersion relation of the fermion is given by

$$\omega_q = \begin{cases} -2h - 2\sqrt{h_{\text{st}}^2 + J^2 \cos^2 q} & \text{for } 0 \leq |q| \leq \frac{\pi}{2} \\ -2h + 2\sqrt{h_{\text{st}}^2 + J^2 \cos^2 q} & \text{for } \frac{\pi}{2} < |q| \leq \pi \end{cases}. \quad (2.5)$$

The above dispersion relation ω_q has an energy gap at $|q| = \pi/2$ for $h_{\text{st}} \neq 0$ (Figure 1). According to the position of the gap, we divide the parameter $h_{\text{st}}-h$ space into three phases A, B and C as shown in Figure 2. At the boundary between phases A and B, the uniform field h satisfies $h = h_{\text{AB}} \equiv h_{\text{st}}$, and at the boundary between phases B and C, h satisfies $h = h_{\text{BC}} \equiv \sqrt{h_{\text{st}}^2 + J^2}$.

Note that the total number of fermions is not a conserved quantity in the system described by the Hamiltonian (2.4). Thus, in the ground state, all of the energy levels with $\omega_q \leq 0$ are occupied by the fermions and the levels with $\omega_q > 0$ are unoccupied (or occupied by holes). This leads to the following definitions of ‘the Fermi wave number q_F ’: $q_F = \pm\pi/2$ in phase A, $q_F = \pm \arccos(-\sqrt{h^2 - h_{\text{st}}^2}/J)$ in phase B and $q_F = \pm\pi$ in phase C.

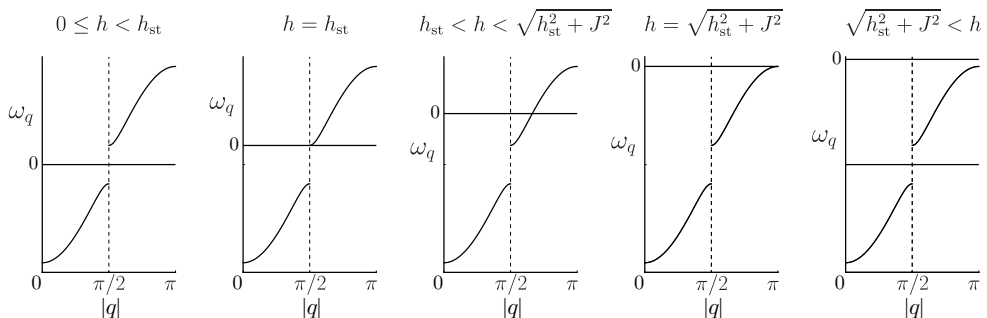


Figure 1: The dispersion relations for $h_{\text{st}} \neq 0$. From left to right: phase A, the boundary between phases A and B, phase B, the boundary between phases B and C, and phase C.

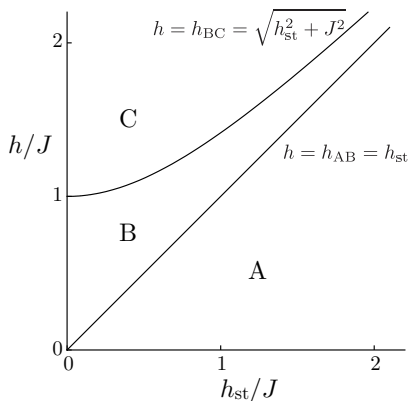


Figure 2: The phase diagram in the $h_{\text{st}}-h$ space. The elementary excitation has an energy gap in phases A and C, whereas it is gapless in phase B.

From the definition of the fermion annihilation and creation operators c_q and $c_q^\dagger$, respectively [17], it is clear that the fermion carries up spin and that the hole carries down spin. In phase A, the total magnetization equals zero in the ground state because one half of the energy levels ($|q| \leq \pi/2$) are occupied by fermions and the other half ($\pi/2 < |q| \leq \pi$) are occupied by holes. The elementary excitation in this phase with $h > 0$ is the one fermion excitation with $q = \pi/2 + 0$, thus, this excitation carries up spin. In phase C, the hole excitation with $q = \pi$ is the elementary excitation. Clearly these elementary excitations have energy gap. In contrast to the above phases, in phase B, there exist gapless fermion- and hole-excitations near the Fermi wave number q_F .

3. Formulation of conductivities

In this section, we consider a general system with a thermal current $\mathcal{J}_Q$ and a spin current $\mathcal{J}_S$, and summarize the formulation of the transport coefficients on the basis of Ref. [11].

Phenomenologically, the thermal and spin currents are caused by the temperature gradient $-\nabla T/T$ and the magnetic field gradient $\nabla\phi_S$, respectively; these relations are written in the following form [9, 10]:

$$\begin{pmatrix} \mathcal{J}_S \\ \mathcal{J}_Q \end{pmatrix} = \begin{pmatrix} L_{SS}(0) & L_{SQ}(0) \\ L_{QS}(0) & L_{QQ}(0) \end{pmatrix} \begin{pmatrix} -\nabla\phi_S \\ -\nabla T/T \end{pmatrix}. \quad (3.1)$$

As is well known, the coefficients $L_{AB}(\omega)$ are given by the Kubo formula:

$$L_{AB}(\omega) = \lim_{\varepsilon \rightarrow +0} \text{Re} \int_0^\infty dt e^{-i\omega t - \varepsilon t} \int_0^{1/T} d\lambda \langle \mathcal{J}_A(-t - i\lambda) \mathcal{J}_B \rangle, \quad (3.2)$$

where $A, B \in \{Q, S\}$, ω is the frequency of the field and $\langle \cdots \rangle$ denotes the thermal average per site.

The spin conduction should be measured under the condition of no temperature gradient, namely $\nabla T = 0$; then, we have

$$\mathcal{J}_S = L_{SS}(0) \cdot (-\nabla\phi_S) = \sigma_S \cdot (-\nabla\phi_S), \quad (3.3)$$

where the spin conductivity σ_S is given by

$$\sigma_S = \lim_{\omega \rightarrow 0} \sigma_S(\omega) = \lim_{\omega \rightarrow 0} L_{SS}(\omega). \quad (3.4)$$

The heat conduction should be measured under the condition of no spin current, namely $\mathcal{J}_S = 0$. Using the Onsager relation $L_{QS}(\omega) = L_{SQ}(\omega)$, we have

$$\mathcal{J}_Q = \left(L_{QQ}(0) - \frac{L_{QS}^2(0)}{L_{SS}(0)} \right) \left(-\frac{\nabla T}{T} \right) = \kappa \cdot (-\nabla T), \quad (3.5)$$

where the thermal conductivity κ is given by

$$\kappa = \lim_{\omega \rightarrow 0} \kappa(\omega) = \lim_{\omega \rightarrow 0} \frac{1}{T} \left(L_{QQ}(\omega) - \frac{L_{QS}^2(\omega)}{L_{SS}(\omega)} \right). \quad (3.6)$$

Generally, the conductivities can be separated into coherent and incoherent parts:

$$\sigma_S(\omega) = \pi D_S \delta(\omega) + \sigma_S^{\text{reg}}(\omega), \quad (3.7)$$

$$\kappa(\omega) = \pi D_Q \delta(\omega) + \kappa^{\text{reg}}(\omega), \quad (3.8)$$

where the coefficients of the delta function are referred to as the Drude weights [7–10]. A non-zero Drude weight implies a ballistic conduction, whereas a zero Drude weight implies a dissipative conduction.

4. Exact derivation of the correlation functions

The time evolution of a local operator A_n , where A_n is the local Hamiltonian or the spin operator associated with the n -th site, is described by the Heisenberg equation:

$$i\frac{\partial A_n}{\partial t} = [A_n, \mathcal{H}]. \quad (4.1)$$

The current operator $j_A(n)$ of the local quantity A_n can be defined such that the above equation satisfies the equation of continuity,

$$\frac{\partial A_n}{\partial t} = -\{j_A(n) - j_A(n-1)\}. \quad (4.2)$$

Thus, the total current operator, $\mathcal{J}_A = \sum_n j_A(n)$, can be defined explicitly [2, 7–11]. In the present model (2.3), the total energy-current operator $\mathcal{J}_E$ and the total spin-current operator $\mathcal{J}_S$ are given by

$$\mathcal{J}_E = \sum_{n=1}^{2N} j_E(n) = \frac{J^2}{2} \sum_{n=1}^{2N} \{\sigma_{n-1}^y \sigma_n^z \sigma_{n+1}^x - \sigma_{n-1}^x \sigma_n^z \sigma_{n+1}^y\} \quad (4.3)$$

and

$$\mathcal{J}_S = \sum_{n=1}^{2N} j_S(n) = \frac{J}{2} \sum_{n=1}^{2N} \{\sigma_n^x \sigma_{n+1}^y - \sigma_n^y \sigma_{n+1}^x\}, \quad (4.4)$$

respectively. It is easy to confirm the following commutation relations for $h_{\text{st}} \neq 0$:

$$[\mathcal{J}_E, \mathcal{H}] = 0 \quad \text{and} \quad [\mathcal{J}_S, \mathcal{H}] \neq 0, \quad (4.5)$$

namely, $\mathcal{J}_E$ is a constant of motion and $\mathcal{J}_S$ is a non-conserved quantity. Note that the heat current $\mathcal{J}_Q$ for $h \neq 0$ is represented by

$$\mathcal{J}_Q \equiv \mathcal{J}_E - 2h\mathcal{J}_S. \quad (4.6)$$

Substituting the above current operators into eq. (3.2), and using eqs. (3.6) and (3.8), we can write the thermal Drude weight D_Q as [7–11]

$$D_Q = \frac{1}{T^2} \left(\langle \mathcal{J}_E^2 \rangle - \frac{1}{T} \frac{\langle \mathcal{J}_E \mathcal{J}_S \rangle^2}{D_S} \right). \quad (4.7)$$

Here, we have used the fact that the time-dependent current operator in eq. (3.2) is defined by $\mathcal{J}(t) \equiv e^{it\mathcal{H}} \mathcal{J} e^{-it\mathcal{H}}$, and $\mathcal{J}_E$ is a constant of motion, namely $[\mathcal{J}_E, \mathcal{H}] = 0$.

Considering the internal energy per site $e(\phi)$ for the wave function $\psi(n)$ with imposition of the twist boundary condition $\psi(n+2N) = e^{2iN\phi} \psi(n)$, we can derive the spin Drude weight as the curvature of this internal energy [1–4, 10, 11]:

$$D_S = \left. \frac{d^2 e(\phi)}{d\phi^2} \right|_{\phi=0} = \sum_m p_m \left. \frac{d^2 E_m}{d\phi^2} \right|_{\phi=0}, \quad (4.8)$$

where p_m is the Boltzmann weight for an eigenstate $|m\rangle$ of the Hamiltonian with an energy eigenvalue E_m .

In order to derive the correlation functions in eq. (4.7) and the spin Drude weight (4.8), we consider the following Hamiltonian

$$\tilde{\mathcal{H}} = U\mathcal{H}U^{-1} - \lambda\mathcal{J}_E, \quad (4.9)$$

where $U \equiv \prod_n e^{in\phi\sigma_n^z/2}$ is the unitary operator and the unitary transformation $U\mathcal{H}U^{-1}$ twists the spin operators around the z axis. Because $[U, \mathcal{H}_{\text{st}}] = [U, \mathcal{H}_Z] = 0$, the first term of $\tilde{\mathcal{H}}$ can be written as follows:

$$\begin{aligned} U\mathcal{H}U^{-1} &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \phi^{2k} \mathcal{H}_0 - \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \phi^{2k+1} \mathcal{J}_S + \mathcal{H}_{\text{st}} + \mathcal{H}_Z \\ &= \cos \phi \mathcal{H}_0 - \sin \phi \mathcal{J}_S + \mathcal{H}_{\text{st}} + \mathcal{H}_Z. \end{aligned} \quad (4.10)$$

Using the Jordan-Wigner transformation and the Fourier transformation, the Hamiltonian $\tilde{\mathcal{H}}$ is diagonalized into the following form:

$$\tilde{\mathcal{H}} = \sum_q \tilde{\omega}_{q-\phi} \left(c_q^\dagger c_q - \frac{1}{2} \right), \quad (4.11)$$

where the dispersion relation $\tilde{\omega}_{q-\phi}$ is given by

$$\tilde{\omega}_{q-\phi} = \begin{cases} -2 \left(h - \lambda J^2 \sin 2q + \sqrt{h_{\text{st}}^2 + J^2 \cos^2(q-\phi)} \right) & \text{for } 0 \leq |q-\phi| \leq \frac{\pi}{2} \\ -2 \left(h - \lambda J^2 \sin 2q - \sqrt{h_{\text{st}}^2 + J^2 \cos^2(q-\phi)} \right) & \text{for } \frac{\pi}{2} < |q-\phi| \leq \pi \end{cases}. \quad (4.12)$$

The logarithm of the partition function and the internal energy per site in the thermodynamic limit $2N \rightarrow \infty$ are represented by the following integrals:

$$\lim_{N \rightarrow \infty} \frac{1}{2N} \log \text{Tr} e^{\tilde{\mathcal{H}}/T} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left(2 \cosh \frac{\tilde{\omega}_{q-\phi}}{2T} \right) dq \quad (4.13)$$

and

$$\lim_{N \rightarrow \infty} \frac{1}{2N} \frac{\text{Tr} \tilde{\mathcal{H}} e^{\tilde{\mathcal{H}}/T}}{\text{Tr} e^{\tilde{\mathcal{H}}/T}} = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\tilde{\omega}_{q-\phi}}{2} \tanh \frac{\tilde{\omega}_{q-\phi}}{2T} dq, \quad (4.14)$$

respectively. Then, the correlation functions and the spin Drude weight in the thermodynamic limit $2N \rightarrow \infty$ are given by

$$\langle \mathcal{J}_E^2 \rangle = \frac{1}{\pi} \int_0^{\pi/2} \sum_{\pm} \frac{J^4 \sin^2 2q}{\cosh^2 \beta(-h \pm \sqrt{h_{\text{st}}^2 + J^2 \cos^2 q})} dq, \quad (4.15)$$

$$\langle \mathcal{J}_E \mathcal{J}_S \rangle = \frac{1}{\pi} \int_0^{\pi/2} \sum_{\pm} \frac{\pm J^4 \sin^2 2q}{2\sqrt{h_{\text{st}}^2 + J^2 \cos^2 q} \cosh^2 \beta(-h \pm \sqrt{h_{\text{st}}^2 + J^2 \cos^2 q})} dq, \quad (4.16)$$

$$D_S = \frac{1}{\pi} \int_0^{\pi/2} \sum_{\pm} \frac{\beta J^4 \sin^2 2q}{4(h_{\text{st}}^2 + J^2 \cos^2 q) \cosh^2 \beta(-h \pm \sqrt{h_{\text{st}}^2 + J^2 \cos^2 q})} dq, \quad (4.17)$$

respectively. Substituting these equations into eq. (4.7), we can calculate the thermal Drude weight for arbitrary temperatures and magnetic fields.

5. Thermal and spin Drude weights

Figure 3 shows the dependence of the thermal Drude weights on the temperature for (a) $h_{\text{st}}/J = 0.4$ and (b) $h_{\text{st}}/J = 0$. In phase B, the temperature dependence of D_Q for $h_{\text{st}} \neq 0$ shows a double peak. Around the temperature $T \simeq |\omega_{\pi/2+0} + \omega_{\pi/2-0}|/2$, thermal excitations decrease due to the energy gap; thus, the thermal conductivity becomes smaller and the double-peak behaviour of D_Q appears.

In the ground state, the thermal Drude weight is zero for the whole parameter regions. The low-temperature behaviour of D_Q for each parameter region is written in the following form:

$$D_Q = \begin{cases} \frac{3J}{2\sqrt{\pi}h_{\text{st}}} T^{3/2} e^{-2(h_{\text{st}}-h)/T} & \text{for } 0 \leq h < h_{\text{st}} \\ \frac{3J\{6(\sqrt{2}-1)\zeta(3/2)^2 - 5(\sqrt{2}-4)\zeta(1/2)\zeta(5/2)\}}{16\sqrt{\pi}h\zeta(1/2)} T^{3/2} & \text{for } h = h_{\text{st}} \\ \frac{2\pi\sqrt{(h^2 - h_{\text{st}}^2)(h_{\text{st}}^2 + J^2 - h^2)}}{3h} T & \text{for } h_{\text{st}} < h < \sqrt{h_{\text{st}}^2 + J^2} \\ \frac{3J\{6(\sqrt{2}-1)\zeta(3/2)^2 - 5(\sqrt{2}-4)\zeta(1/2)\zeta(5/2)\}}{16\sqrt{\pi}h\zeta(1/2)} T^{3/2} & \text{for } h = \sqrt{h_{\text{st}}^2 + J^2} \\ \frac{3J}{2\sqrt{\pi}\sqrt{h_{\text{st}}^2 + J^2}} T^{3/2} e^{-2(h - \sqrt{h_{\text{st}}^2 + J^2})/T} & \text{for } \sqrt{h_{\text{st}}^2 + J^2} < h \end{cases}, \quad (5.1)$$

where $\zeta(z)$ denotes the Riemann zeta function. Because of the elementary excitation gap in phases A and C, the thermal Drude weights at low temperatures have exponentially small values. On the other hand, in the gapless phase B, the dependence of D_Q on the temperature behaves linearly at low temperatures.

The spin Drude weight has a non-zero value at the finite temperatures in the whole parameter regions as shown in Figure 4, although the spin current is a non-conserved quantity. Reflecting the ground state behaviour of the spin Drude weight, the low temperature behaviour of D_S becomes complicated. The asymptotic form of the spin Drude weight for small T is written as follows:

$$D_S = \begin{cases} \frac{J}{\sqrt{\pi}h_{\text{st}}} \sqrt{T} e^{-2(h_{\text{st}}-h)/T} & \text{for } 0 \leq h < h_{\text{st}} \\ \frac{(1-\sqrt{2})J\zeta(1/2)}{\sqrt{\pi}h} \sqrt{T} & \text{for } h = h_{\text{st}} \\ \frac{2\pi\sqrt{(h^2 - h_{\text{st}}^2)(h_{\text{st}}^2 + J^2 - h^2)}}{\pi h} & \text{for } h_{\text{st}} < h < \sqrt{h_{\text{st}}^2 + J^2} \\ \frac{(1-\sqrt{2})J\zeta(1/2)}{\sqrt{\pi}h} \sqrt{T} & \text{for } h = \sqrt{h_{\text{st}}^2 + J^2} \\ \frac{J}{\sqrt{\pi}\sqrt{h_{\text{st}}^2 + J^2}} \sqrt{T} e^{-2(h - \sqrt{h_{\text{st}}^2 + J^2})/T} & \text{for } \sqrt{h_{\text{st}}^2 + J^2} < h \end{cases}. \quad (5.2)$$

In the gap phases A and C, the low temperature behaviour of D_S has the same exponential factor as that of D_Q . In contrast to the thermal Drude weight, the spin Drude weight in

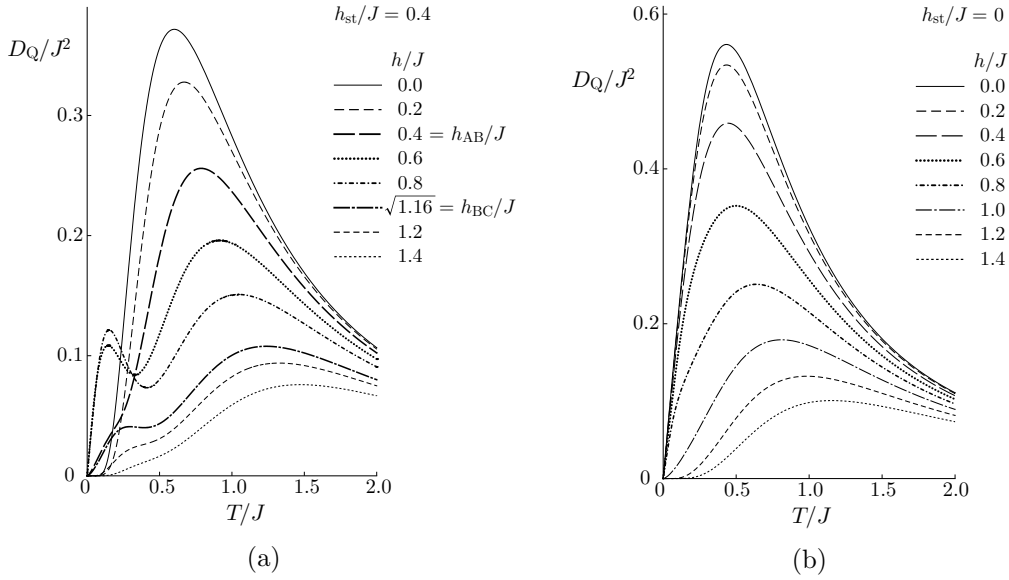


Figure 3: Dependence of the thermal Drude weight on temperature for (a) $h_{st}/J = 0.4$ and (b) $h_{st}/J = 0$.

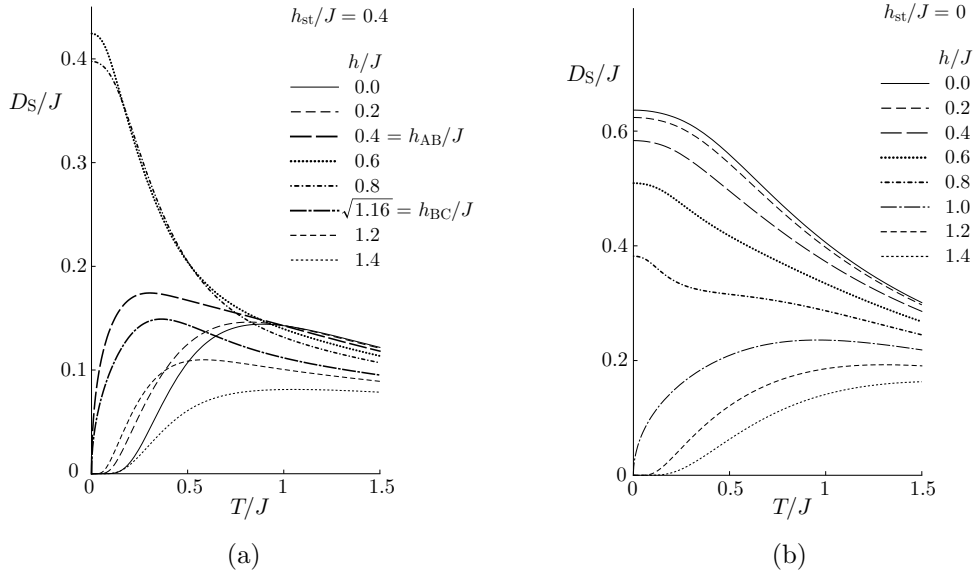


Figure 4: Dependence of the spin Drude weight on temperature for (a) $h_{st}/J = 0.4$ and (b) $h_{st}/J = 0$.

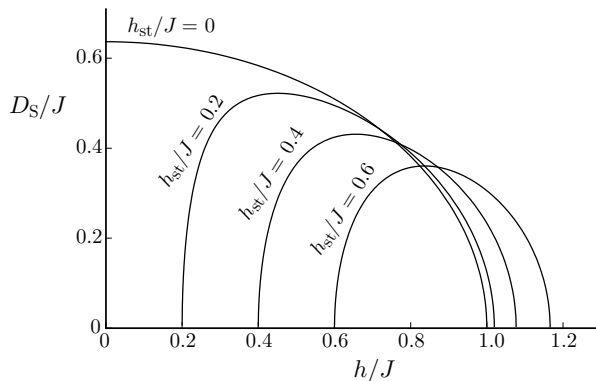


Figure 5: Spin Drude weights in the ground states for $h_{\text{st}}/J = 0, 0.2, 0.4$ and 0.6 .

the ground state has a non-zero value in the gapless phase B. The ground-state spin Drude weights for several values of the staggered field are shown in Figure 5.

6. Magnetic Seebeck effect

The correlation effect between the thermal and spin currents, namely the magnetothermal effect, is contained in the off-diagonal elements of the transport-coefficient matrix (3.1). For $h = 0$, the spin-reversal symmetry leads to no correlation, even for a finite staggered field. The uniform magnetic field breaks the spin-reversal symmetry, then, the magnetothermal effect appears. The temperature gradient is transformed into the potential energy of the fermion taking on the role of the spin conduction. In analogy to the Seebeck effect for electron systems, we refer to this effect as the magnetic Seebeck effect [11]. This effect can be represented by the magnetic Seebeck coefficient defined by $S \equiv -\nabla\phi/\nabla T$. This quantity should be measured under the condition of no spin current, thus, S can be written as

$$S = -\frac{\nabla\phi}{\nabla T} = -\frac{1}{T} \frac{L_{QS}}{L_{SS}} = -\frac{1}{T} \frac{\langle \mathcal{J}_Q \mathcal{J}_S \rangle / T}{D_S} = -\frac{1}{T} \left\{ \frac{1}{T} \frac{\langle \mathcal{J}_E \mathcal{J}_S \rangle}{D_S} - 2h \right\}. \quad (6.1)$$

The sign of the magnetic Seebeck coefficient indicates a crucial criterion for clarifying the types of carriers. That is, the carriers of the transport are dominated by up-spin carriers (fermions) if the sign of S is positive, whereas it is dominated by down-spin carriers (holes) if the sign of S is negative.

Substituting eqs. (4.16) and (4.17) into eq. (6.1), we can calculate the magnetic Seebeck coefficient for arbitrary temperatures and magnetic fields. Figure 6 shows the dependence of the magnetic Seebeck effect on the temperature for (a) $h_{\text{st}}/J = 0.4$ and (b) $h_{\text{st}}/J = 0$. For $h = 0$, the magnetic Seebeck coefficient is always zero because of the spin reversal symmetry. For $h_{\text{st}} = 0$ and $h > 0$ (Figure 6 (b)), the hole excitation is relatively larger than the fermion excitation, therefore, S always takes negative values and the main carriers of spin current are

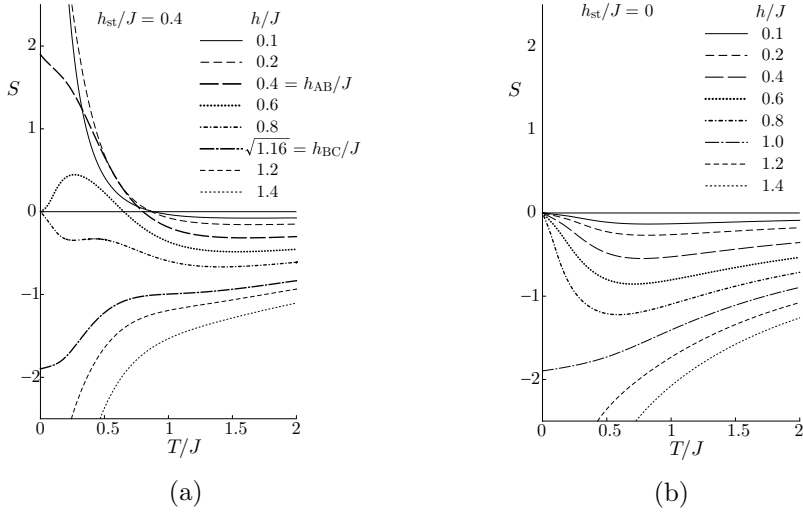


Figure 6: Dependence of the magnetic Seebeck coefficient on temperature for (a) $h_{\text{st}}/J = 0.4$ and (b) $h_{\text{st}}/J = 0$.

holes (down-spin carriers).

For $h_{\text{st}} \neq 0$, as shown in Figure 6 (a), the low temperature behaviour of the magnetic Seebeck coefficient varies depending on the excitation spectrum. In the gapless phase B, S becomes zero at zero temperature and shows the linear T dependence

$$S \simeq \frac{\pi^2(h_{\text{st}}^4 + h_{\text{st}}^2 J^2 - h^4)}{6h(h^2 - h_{\text{st}}^2)(h_{\text{st}}^2 + J^2 - h^2)} T \quad (6.2)$$

in the sufficiently low temperature region. From eq. (6.2), we can see that the sign of S at low temperature is positive for $h^4 < h_{\text{st}}^4 + h_{\text{st}}^2 J^2$ and negative for $h^4 > h_{\text{st}}^4 + h_{\text{st}}^2 J^2$. This sign corresponds to the sign of the second derivative of the dispersion relation (2.5) at the Fermi wave number q_{F} . For $h^4 < h_{\text{st}}^4 + h_{\text{st}}^2 J^2$, $\partial^2 \omega_q / \partial q^2|_{q=q_{\text{F}}}$ is negative, thus the number of the low lying fermion excitations become relatively larger than that of the hole excitations. As a result, the low-temperature S takes positive values in this parameter region.

The elementary excitation is the fermion excitation in phase A and the hole excitation in phase C. Therefore, the magnetic Seebeck coefficient becomes positive in phase A, and negative in phase C at sufficiently low temperature. In these gap phases, S diverges at the zero-temperature with the following asymptotic form:

$$S \sim \pm \frac{\Delta}{T}, \quad (6.3)$$

where Δ denotes the elementary excitation gap, namely $\Delta = 2(h_{\text{st}} - h)$ in the phase A and $\Delta = 2(h - \sqrt{h_{\text{st}}^2 + J^2})$ in the phase C.

At the phase boundaries between phases A and B and between phases B and C, the magnetic Seebeck coefficients converge to the values

$$\lim_{T \rightarrow 0} S = \pm \frac{3(2 - \sqrt{2}) \zeta(3/2)}{4(1 - \sqrt{2}) \zeta(1/2)} = \pm 1.89738 \quad (6.4)$$

at zero temperature. These values are independent of the staggered magnetic field.

7. Conclusion

In the present study, we have exactly calculated the thermal and spin Drude weights of the XX spin chain in the staggered and uniform magnetic fields. We have found that the energy gap caused by the staggered magnetic field leads to double peak behaviour of the thermal Drude weight. The low-temperature asymptotic forms of the thermal and spin Drude weights are derived. In the ground state, the Drude weights become zero except for the spin Drude weight in the gapless phase B. From the analysis of the magnetic Seebeck coefficient, we have found that the spin direction of the main carrier of the currents varies depending on the staggered field. In particular, the sign of the low-temperature magnetic Seebeck coefficient changes from positive to negative as h increases at the boundary $h^4 = h_{\text{st}}^4 + h_{\text{st}}^2 J^2$. This boundary corresponds to the point at which the curvature of the dispersion relation at the Fermi wave number equals zero.

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